(Attention : this is an abbreviation of my working paper in Chinese Central Bank, so some contents of the original paper cannot be shown based on the regulation)

Using Shapley's asymmetric power index to measure Chinese commercial banks' contribution to the systemic risk

Zhuo Chen

Abstract

An individual commercial bank can tip the banking system from a state of stability to a state of instability if its losses in response to shocks push losses for the system as a whole above a critical threshold. Using the Shapley value – a concept generated in game theory originally, I calculate an individual bank's contribution to the systemic risk. As previous studies have only discussed the primary theory of the method without application to enormous bank data, I take Chinese commercial banks for an example and evaluated what is needed to apply the approach to bank data so that it could be used as a risk assessment tool.

1 Introduction

How to regulate banks in ways that reflect the different contributions banks make to systemic risk in the financial system has long been concerned by policymakers. One aspect of how an individual bank's failure could contribute to systemic risk could be defined in terms of whether its failure is considered to be pivotal in tipping the banking system from a state of stability to a state of instability. Based on this idea, an approach is developed by Rodney J Garratt (2012) that can be used to calculate the marginal contributions of individual bank failures to systemic risk.

The approach is based on a measure originally introduced by the mathematician and economist Lloyd Shapley. The so-called Shapley value is a way of allocating the output produced by a group among its members in a way that reflects fairly their individual contributions. In their paper, the Shapley value is applied to the situation where the group is a set of banks that fail due to shocks to the values of their assets.

The approach includes two key variables: the values of banks' exposures to different asset classes; and the levels of banks' capital available to absorb losses on their asset holdings. The banking system can be hit by a range of shocks, which are defined in terms of the extent to which they reduce the value of the different asset classes. The shocks are assumed to occur with equal probability. For each possible shock, banks can be lined up in the order that they would fail as a result of that shock. The pivotal bank is the one that, when it is added to the banks that fail before it, causes the value of the failed banks' assets to move above a critical threshold value – this is defined as a systemic event. The pivotal bank receives a score of one (and other banks receive a score of zero). Taking an average of a bank's score over the range of possible shocks can lead to a measure of a bank's contribution to systemic risk.

Asymmetric power indices for a system of banks can be calculated by first defining banks' positions in an *l*-dimensional 'leverage' space, where *l* denotes the number of distinct asset classes. Positions are defined in terms of leverage – asset exposures relative to capital – because the ordering of bank failures in response to a particular type of shock ought to depend not only on the sizes of banks' exposures to that shock, but also on their ability to withstand it.

The ordering of bank failures is determined by random shocks to the value of assets. A particular shock is represented by a direction in the leverage space. For each type of shock, there is an implied order in which banks would fail. This is based in part on the proximity of the asset position to the shock (banks with asset mixes that are similar to the shock fail first) and in part on the distance of the asset position from the origin (banks with high ratios of assets to capital fail first). These two aspects determine the overall ordering of bank failures. Given a probability distribution over shocks, a probability distribution was generated over different orderings of bank failures. This in turn is used to compute the power index for each bank.

Rodney's research(2012) has only considered a two-dimensional leverage space, so each bank is therefore represented as a point in a two dimensional Euclidean space with coordinates defined by the ratio of holdings of each asset class to capital. Meanwhile, he has simply discussed a fictitious banking system including 3 banks. While in this paper, I extend the approach to higher degree like 3 or 5 as well as apply the approach to Chinese commercial bank's data without a limitation of the number of banks. It means that I take the approach into practice. Combining the practical situation of the Chinese commercial banking system, I divide the whole banks in to three groups: the 4 state-owned commercial banks, listed commercial banks, and other commercial banks. For each group, I separately discuss an individual bank's contribution to systemic risk in a two, three & five dimensional Euclidean space. An analysis is made based on the outcome and I present several improvement and extension to the approach.

This abbreviation of the paper proceeds as follows. Section 2 reviews Rodney's research which describes how the effect of bank failures on systemic risk is measured using the Shapley's asymmetric power index. And I quote his three bank example to better illustrate the primary model. Section 3 presents my extensions of the approach. I apply the approach to Chinese commercial banks and extend the approach to higher degree like 3 or 5, which discusses the practical application of the approach. Section 4 concludes.

2 Primary Model

2.1 Mathematical Theory - Calculating banks' contributions to systemic risk

In the primary model with the degree of 2, the set of banks is denoted by *N*={1,…,n}, and banks' assets are divided into two classes, domestic (H_i) and foreign (F_i). Each bank is represented as a point in a two-dimensional Euclidean leverage space with coordinates defined by the ratio of each type of asset to capital (C_i). That is, the position of bank $\mathrm{i} \in \mathrm{N}$ is a point $a_i = (h_i + f_i) \in \mathbb{R}_+{}^2$, where $h_i = H_i/C_i$ and $f_i = F_i/C_i$. Types of asset shocks are characterised by two-dimensional, unit-length, vectors $z \in \mathbb{R}_+{}^2$ that span the two-dimensional leverage space $\{h_i, f_i\}$, centred on the origin. To determine the order of failure for a given shock and a given vector of bank positions we follow Shapley (1977) and assume that bank *i* fails before bank *j* if:

$$\mathrm{z} \cdot a_i > \mathrm{z} \cdot a_j \tag{1}$$

The order of bank failures is therefore determined by the relationship between the banks' positions in the leverage space and the type of asset shock. In particular, equation (1) holds if and only if $\| a_i \| cos\theta_i > \| a_j \| cos\theta_j$, where $\| a_k \|$ denotes the distance of a_k from the origin and θ_k denotes the angle between a_k and z. So, holding the angle between its position and the shock fixed, a bank will fail earlier in the overall ordering if its capital is decreased (i.e. $\| a_k \|$ is increased). Likewise, holding the distance from the origin fixed, a bank fails earlier if the direction implied by their asset mix is closer to the direction of the shock (i.e. the angle θ_k is smaller). Note that equation (1) implies that the order of arrivals of bank failure will be the same for any shock z that lies on the same ray from the origin. Shocks can therefore be distinguished only by their direction, not their magnitude.

In this part, each bank's power index is measured by the proportion of shocks for which its failure is pivotal in causing a systemic event. A bank failure is pivotal when its inclusion in a group of already-failed banks causes a systemic event. I define a systemic event to be one in which the total assets of failed banks $\Sigma_i w_i$ exceeds a pre-defined fraction of banking system assets ξ, where the vector of weights $w = \{w_1, \dots, w_n\}$, reflects banks' shares of total system assets:

$$w_i = \frac{H_i+F_i}{\Sigma_{i\in N}(H_i+F_i)} \tag{2}$$

Each subgroup of banks $G \subseteq N$ has a value $\mathrm{v(G)} = 1$ if $\Sigma_i w_i > \xi$ and $\mathrm{v(G)} = 0$ otherwise. This is our so-called 'characteristic function'.

Let π denote an ordering of the banks (i.e. an order in which banks fail). For each bank $\mathrm{i} = 1, \ldots, \mathrm{n}$ let $\mathrm{p}_\pi^i = \{j: \pi(i) > \pi(j)\}$ denote the set of players preceding *i* in the order π. The marginal contribution of bank *i* in order π is $\mathrm{v}(\mathrm{p}_\pi^i \cup i) - \mathrm{v}(\mathrm{p}_\pi^i)$.

We assume that asset shocks occur randomly with respect to a uniform distribution over all possible shocks (i.e. over all vectors $z \in \mathbb{R}_+{}^2$). Then, for each possible order π of banks, we can compute the probability $\theta(\pi)$ that the random shock z generates the order π.

The contribution of each bank's failure to systemic risk is given by:

$$\phi(\mathrm{v}; \mathrm{H}, \mathrm{F}, \mathrm{C}) = \Sigma_{\pi\in\Pi}\theta(\pi)[\mathrm{v}(\mathrm{p}_\pi^i \cup i) - \mathrm{v}(\mathrm{p}_\pi^i)] \tag{3}$$

The term in square brackets in equation (3) takes a value of one or zero. These risk measures have the property that $\Sigma_i \phi_i = 1$.

2.2 Three-Bank example

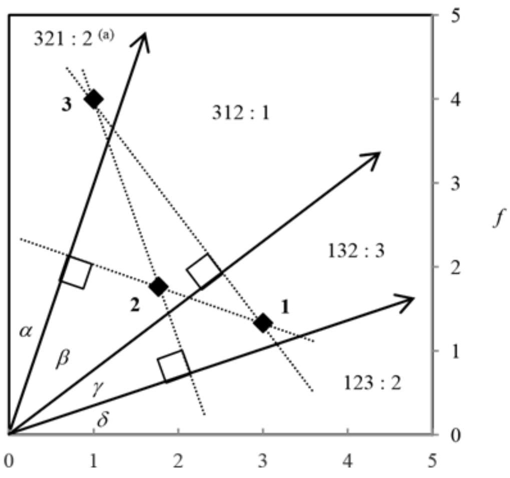
This section illustrates how the power index for each bank is calculated, for a system of three banks. Suppose banks' asset holdings are as specified in Table 1 and that banks' positions in a two-dimensional leverage space are as shown in Chart 1 with coordinates (h_i, f_i). Types of asset shock are represented by directional arrows. It is straightforward to show that the ordering implied by equation (1) associated with any shock can be determined by dropping perpendiculars from the points to the shafts of the directional arrows. Perpendiculars that lie higher up the shaft of an arrow denote earlier failure according to equation (1). Moreover, as we rotate the arrows through the asset space, the order in which banks fail only changes when the arrow crosses a perpendicular associated with one of the sides of the triangle defined by the three asset positions.

The orderings generated by rotating an arrow 90 degrees through the asset space are shown in Chart 1. Consider a shock in the direction of the *f*-axis. Asset positions with high values of *f* relative to *h* imply the ordering 3,2,1. As that arrow rotates clockwise, it becomes perpendicular to the line through points 1 and 2 and the order of failure switches to 3,1,2. Further along the rotation it becomes perpendicular to the line through points 1 and 3 and the order switches to 1,3,2. Finally, for asset positions with low values of *f* relative to *h* the ordering is 1,2,3.

Table 1: A Three-bank Example

	Bank		
	1	2	3
Domestic assets (H_i)	90	30	20
Foreign assets (F_i)	40	30	80
Capital (C_i)	30	17	20
h_i	3	1.765	1
f_i	1.333	1.765	4
Weight (w_i)	0.448	0.207	0.345
Strength $((H_i + F_i)/C_i)$	4.333	3.529	5
Diversification $(\min\{H_i, F_i\}/(H_i + F_i))$	0.308	0.5	0.2

Chart 1: Illustration of a three-bank example



To determine each bank's power index, I compute the fraction of shocks for which each bank failure is pivotal, in the sense that it causes the value of the characteristic function $v(G)$ to change from 0 to 1. With a chosen risk threshold of $\xi = 0.5$, in this example, the pivotal bank failure will always be the second bank to fail, as indicated by the number after the colon in Chart 1. The power indices for banks 1, 2 and 3 are given by $\beta/90$, （$\alpha+\delta$）$/90$ and $\gamma/90$, respectively. Using the positions stated in Table 1, the angle values are $\alpha = 19.3$, $\beta = 33.9$, $\gamma = 18.1$, and $\delta = 18.7$. The power indices for banks 1, 2 and 3 are therefore given by $\phi(v; H, F, C) = (0.38, 0.42, 0.20)$. In contrast to a standard (symmetric) power index, not all orderings are equally likely and some orderings are ruled out completely.

Bank 2 has the highest power index (largest ϕ_i) because its failure is pivotal in terms of causing a systemic event more often than either of the failure of the other two banks. Further inspection of Chart 1 indicates why this is the case. Bank 2 is centrally located relative to banks 1 and 3, reflecting its more diversified asset base. Asset shocks that are concentrated in the domestic sector bring down bank 1 first, because bank 1 has a large share of domestic assets on its balance sheet. Conversely, asset shocks that are concentrated overseas bring down bank 3 first because bank 3 has a large share of foreign assets on its balance sheet. In both cases, however, bank 2 is positioned to fail second and hence its power index is equal to the 42 per cent share of the asset space covered by these shocks. But notice that bank 2 is not the first to fail when asset shocks involve broadly even mixtures of domestic and foreign assets: both asset mix and balance sheet strength matter in determining the order of failures. Bank 2 has a diversified mix of domestic and foreign assets, but it is also well capitalized (it has a ratio of total assets to capital of 60/17=3.529) relative to its counterparts (4.333 and 5 for banks 1 and 3 respectively). Bank 2 is therefore relatively less likely to fail across possible vectors of asset shocks. It is only in extreme cases, where asset shocks fall almost entirely on one asset class or the other that one of the other banks is less likely to fail.

3 Application & Extensions

3.1 Empirical Analysis – Chinese commercial banks

As has been presented above, it is succinct and straightforward to measure the banks' contributions to systemic risk in a three-bank example with assets divided into two classes. However, in practice, there are many banks in a real financial market and their assets are usually divided into more classes like three or five. Therefore, I will discuss how to apply the method to a system with many diversified banks in this part as well as extend the method to higher degree in the next part, in both of which I take the system of Chinese commercial banks as a practical instance.

To begin with, as is known to all, there are over two hundred commercial banks in China. However, there is a big difference between some two banks like Bank of China and Bank of Hanyang. The total asset of the former is nearly one thousand times that of the later. It is obvious that the failure of Bank of China has much more significance than Bank of Hanyang, which means that considering them in a same group is meaningless. Since some big banks' assets account for much of the total assets of the commercial banking system, it is easy for the total assets of failed banks exceeds a pre-defined fraction of banking system when a big bank is added to the list of failed banks, in which case most of the other relatively smaller banks will make no contribution to the systemic risk using this approach. However, considering the different main business and customers faced, many smaller banks except for big state-owned commercial banks also plays a crucial role in some parts of the banking system and the everlasting failures of the similar banks can also make a crisis in the financial market or cause a systemic event.

Therefore, based on the analysis above and the measuring process in section 2, I divide all of the commercial banks into three groups: the 4 state-owned commercial banks, listed commercial banks, and other commercial banks. For each group, I separately discuss an individual bank's contribution to systemic risk in a two-dimensional space in this section (the situation of higher degree is discussed in section 3.2).

While the approach to calculate the bank's contribution to systemic risk in the primary three-bank model is only accessible when the number of the bank is very limited, to find an improved method which can contain as many as possible numbers of banks is of significance. The following is my approach.

Firstly, I divide the banks' assets into 2 classes and calculate the slope of the two points which represent the two banks. Later, I obtain the slope of the unit-length direction vector which is perpendicular to the link of the two points. If the slope of the direction vector from the origin is positive, thus the space spanned by the asset class were divided into 2 parts by the vector. In the left part of the space, the point with bigger f_i will always fail earlier than the other point. That means the shock of the left part of the space has more influence on this point with bigger f_i than the other point. Then the point with bigger f_i receives a score of 1 in left part and 0 in right part. The other point scores just the opposite. If the slope of the direction vector from the origin is not positive, then the the point with bigger f_i receives a score of 1 in the whole space while the other receives 0. Then each point will receive a score of one or zero according to the comparison, thus the space spanned by the asset class are divided into finite partial sections where the order of the banks failed depends on their total scores in this section. Adding the assets of the failed banks by the order determined before, I get the pivotal bank which make the total assets of failed banks exceed the pre-defined fraction in each section. For each bank, its final Shapley value is determined by adding the angles of the section in which it is the pivotal bank and then dividing the sum of the angles by 90. All processes of the method are realized by programming with Matlab and there is no limitation of the number of the banks.

The banks' contribution to systemic risk are respectively measured in the following steps and I make some analysis of the outcome. Combining the core profitable businesses of the commercial banks, in the situation of 2 degree or in this section. I divide the assets of banks into two classes：security-related assets & mortgage-related assets. Since this paper is just an abbreviation of my working paper in PBC (People's Bank of China), I choose to present only the analysis of the data in 2013. But it is enough to reflect my core point of view.

3.1.1 4 state-owned commercial banks

The first group is made up with 4 famous state-owned commercial banks in China: BOC (Bank of China), CCB (China Construction Bank), ICBC (Industrial and Commercial Bank of China), and ABC (Agricultural Bank of China). The common feature of the 4 banks is that their assets are all over ten trillion yuan which is at least ten times that of other commercial banks. The sum of their assets exceeds the sixty percent of the total assets of the commercial banking system. Therefore, it is necessary to put the four banks in a same group to measure their contribution to the systemic risk on their level. Taking the regulation on commercial banks, I choose 0.4 as a critical threshold firstly in this section. The four banks' asset holdings are shown in Table 2. Meanwhile, since the number of the banks in this group is relatively small, I am able to present the orderings generated by rotating an arrow 90 degrees through the asset space in Chart 2.1. In addition, the final Shapley Value or the contribution to systemic risk of each bank is shown in Chart 2.2.

Table 2: Four state-owned commercial banks

	Bank			
	BOC(1)	CCB(2)	ICBC(3)	ABC(4)
Security-related assets (S_i)	2.39	3.53	4.35	3.37
Mortgage-related assets (M_i)	11.49	11.84	14.56	11.19
Capital (C_i)	0.96	1.07	1.28	0.84
s_i	2.48	3.28	3.41	3.99
m_i	11.95	11.02	11.39	13.25
Weight (w_i)	0.22	0.24	0.30	0.23
Strength ($(H_i + F_i)/C_i$)	14.43	14.30	14.80	17.24
Diversification ($\min\{H_i, F_i\}/(H_i + F_i)$)	0.172	0.229	0.230	0.232

Chart 2.1: Illustration of Four state-owned commercial banks (ξ = 0.4)

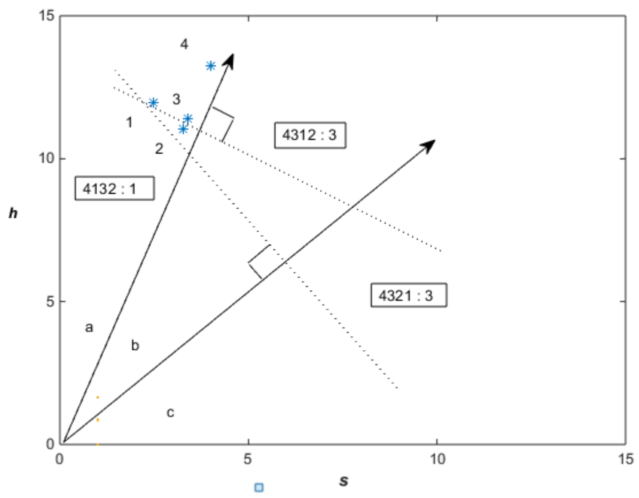
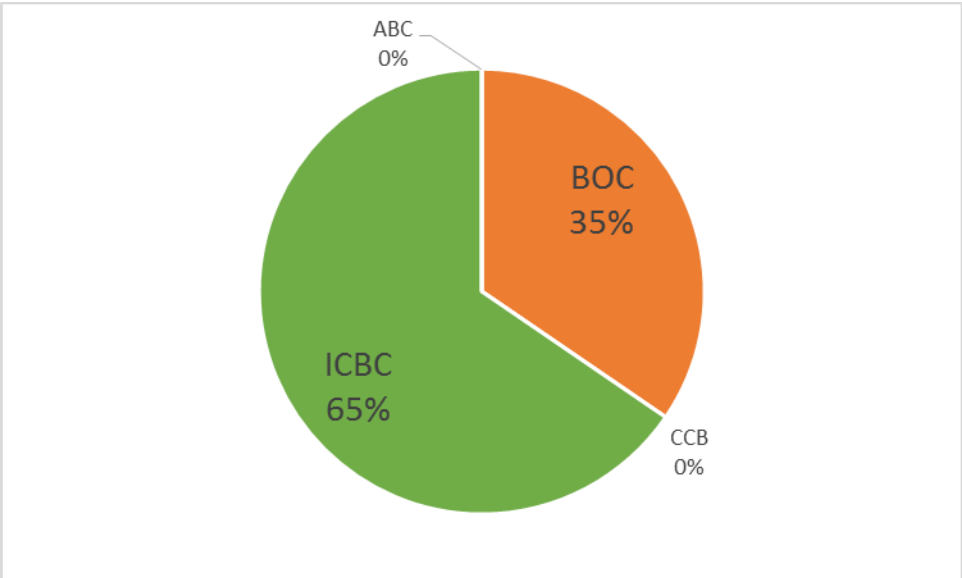


Chart 2.2: Contribution of Four state-owned commercial banks (ξ = 0.4)



ICBC has the highest power index (largest ϕ_i) because its failure is pivotal in terms of causing a systemic event more often than either of the failure of the other 3 banks. Further inspection of Chart 2.1 indicates why this is the case. ICBC is centrally located relative to BOC and CCB, reflecting its more diversified asset base. Asset shocks in all sector bring down ABC first, because ABC has a larger leverage assets on its balance sheet. Based on each bank's asset share of total system assets and the critical threshold set before, the pivotal bank will always be the second failed bank in a shock. That's why ICBC accounts much for the systemic risk.

It has been mentioned above that the distribution of bank's contribution to systemic risk may be instable with a changing critical threshold. Although the critical threshold is usually determined by policy makers with their own tolerance of systemic risk which is related to various economics variables, there is still a need to observing whether the distribution is relatively stable as the critical threshold changes. Therefore, I present the contribution to systemic risk of the four state-owned commercial banks with a ξ =

0.3 and $\xi = 0.45$. (the ξ between 0.3-0.5 is reasonable in my eyes).

Chart 2.3: Contribution of Four state-owned commercial banks ($\xi = 0.3$)

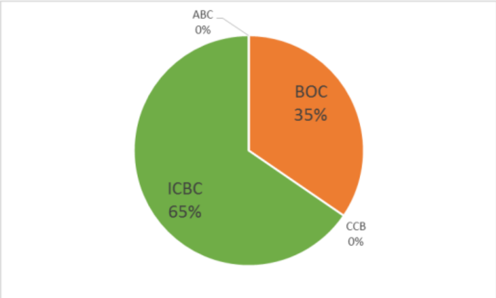
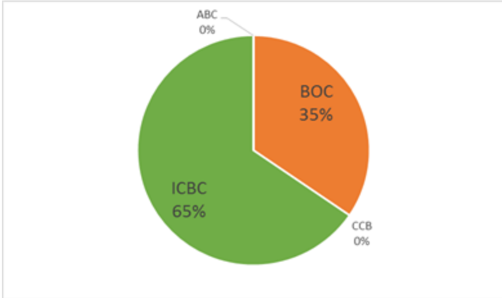


Chart 2.4: Contribution of Four state-owned commercial banks ($\xi = 0.45$)



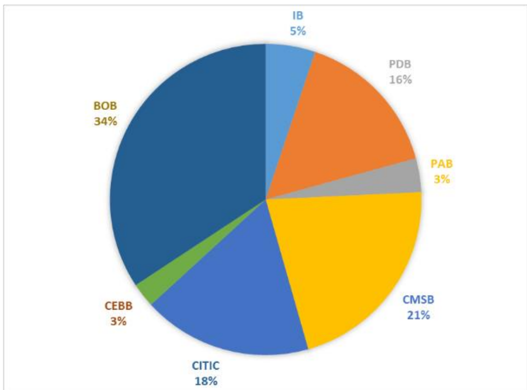
The 2 charts above shows that the distribution of banks' contribution to systemic risk remains stable as the critical threshold changes between the scopes. Therefore, the policy makers of Chinese banking system need to establish a more strict regulation for ICBC & BOC in order to maintain a stable banking system.

3.1.2 Listed commercial banks

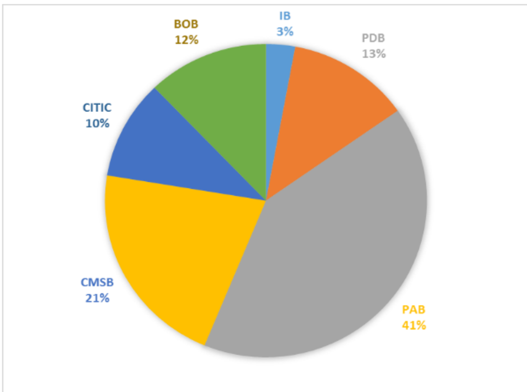
As has been mentioned before, in this part, I will measure the contribution to systemic risk of other 12 listed commercial banks in China except the 4 state-owned commercial banks. The 12 commercial banks are Industrial Bank, BANK OF COMMUNICATIONS, Shanghai Pudong Development Bank, Ping An Bank, China Minsheng Banking, China Merchants Bank, CHINA CITIC BANK, CHINA EVERBRIGHT BANK, Hua Xia Bank, Bank of Beijing, Bank of Nanjing, Bank of Ningbo. The common feature of the 12 banks is that their assets are all over one trillion yuan and the sum of their assets exceeds the twenty percent of the total assets of the commercial banking system. Chart 3 shows their contribution to systemic risk with $\xi = 0.3$, 0.4 , 0.45 or 0.38.

Chart 3 Contribution of 12 listed commercial banks (with different ξ)

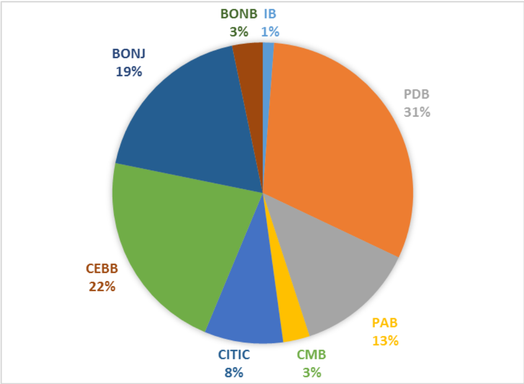
$\xi = 0.3$



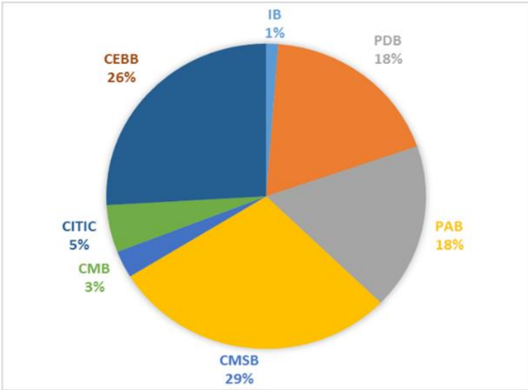
$\xi = 0.38$



ξ = 0.4



ξ = 0.45



It can be seen from the charts above that for this group the contribution of each individual bank varies a lot with a changing critical threshold. Sometimes it is normal because at least in a relatively small interval of ξ . However, in this section, the distribution of contribution of ξ = 0.4 is nearly totally different from that of ξ = 0.42, which means even in a small difference of 0.02 the contribution can change a lot. Such phenomena may make it impossible for the policy maker to make sure of an accurate critical threshold. There is a same problem with the third group with even more commercial banks. (The third group is not discussed in this abbreviation since the second group has already been able to represent most of my creativity of the approach). Therefore, some improvements and extensions have to be made and I discuss them in the section 3.2

3.2 Extension — Higher degree (3 & 5)

As is known to us all, the classification of the assets of commercial banks in financial practice is more than 2 degrees in many times. To gain a better application effect requires us to measure the commercial banks' contributions on a higher degree. Combined with the practical situations in Chinese banking system, there are two classifications need to be discussed individually. The first is to divide the assets of commercial banks into 3 classes based on accounting standard: current assets, long-term assets, & other assets. The second method divides the assets into 5 classes based on the requirement of commercial banks' operation management: cash assets, loan assets, securities, fixed assets & rate assets. Due to the limitation of length, I only discuss the situation with the assets divided into 3 classes in this abbreviation. In fact, the discussion of the two kinds of classification use the same methods as follows.

Looking back to the method that I use to measure the contribution in a two-dimensional leverage space, it is easy to find that the core is to find a direction vector that divide the space in two parts for any 2 points. It is natural to think about finding the same "segregation" for any 2 points in a third-dimensional leverage space. However, it can be proved that the "segregation" in a third-dimensional space can only be a plane containing the origin. It takes enormous calculation to find the "segregation plane" and to measure the space generated by the intersection of different planes. Meanwhile, it is nearly impossible to transplant the method to the space of 5 degree. Therefore, I choose Monte Carlo method to solve the high degree problem.

Monte Carlo method

The core thought of Mont Carlo method is to simulate the whole process through some "experiment" to obtain the frequency of a certain event which can be regarded as the actual probability of the event if the number of the "experiment" is large enough in some circumstances. Therefore, in our problems, the shocks can be regarded as the "experiment" in Monte Carlo method. I can record the order of failed banks and thus find the related pivotal bank in each "experiment" (shocks). Taking all of the shocks into consideration, I can obtain the frequency of a certain bank becoming a pivotal bank in a certain shock. With the number of "experiments" increasing constantly, the frequency will approach the accurate probability as much as possible. Thus I gain a good approach of the distribution of banks' contributions to systemic risk, which can be used to calculate the final Shapley value. It is worth noting that this method has no limitation of the number of the degree, which means I can apply the same method to situations with different degrees as has been mentioned former.

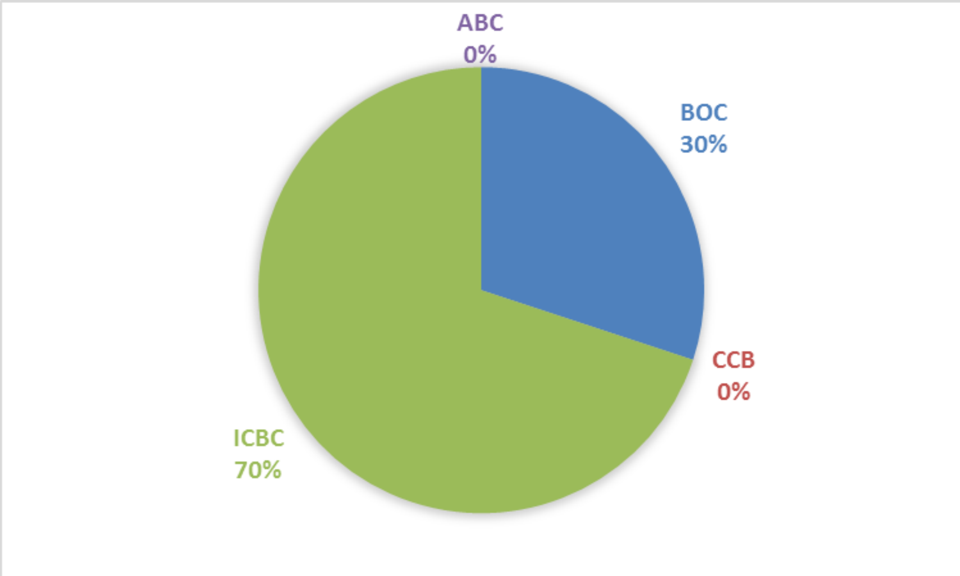
4 state-owned commercial banks

Choosing 0.4 as a critical threshold firstly in this section. The four banks' asset holdings are shown in Table 4. In addition, the final Shapley Value or the contribution to systemic risk of each bank in a three-dimension space is shown in Chart 4.1.

Table 4 : Four state-owned commercial banks in 3-dimension space

	Bank			
	BOC(1)	CCB(2)	ICBC(3)	ABC(4)
Current assets (CA_i)	11.31	12.09	14.48	11.37
Long-term assets (L_i)	2.14	3.06	3.84	2.56
Other assets (O_i)	0.43	0.22	0.59	0.63
Capital (C_i)	0.96	1.07	1.28	0.84
ca_i	11.76	11.25	11.33	13.47
l_i	2.23	2.85	3.01	3.03
o_i	0.44	0.20	0.46	0.75
Weight (w_i)	0.22	0.24	0.30	0.23
Strength ($(CA_i + L_i + O_i)/C_i$)	14.43	14.30	14.80	17.24
Diversification ($\frac{\max\{CA_i,L_i,O_i\}-\min\{CA_i,L_i,O_i\}}{(CA_i+L_i+O_i)}$)	0.784	0.773	0.734	0.738

Chart 4.1 Contribution of Four state-owned commercial banks (3 degree)



As can be shown above, the distribution of contribution to systemic risk is a little different from that of the 2 degree. ICBC accounts more for the systemic risk while BOC less. ABC & CCB remain the same. The difference is easy to understand because I measure the risk exposures of banks assets and shocks in the market from a different angle. The situation with ξ = 0.3 and ξ = 0.45 is in the following.

Chart 4.2: Contribution of Four state-owned commercial banks (ξ = 0.3, 3 degree)

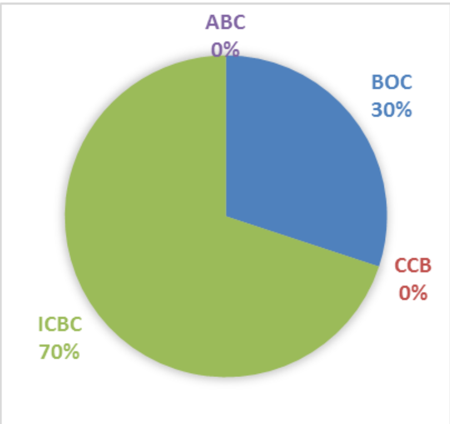
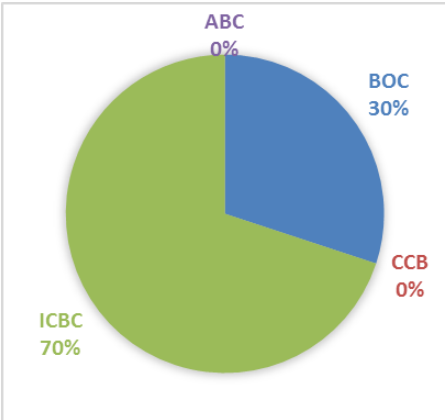


Chart 4.3: Contribution of Four state-owned commercial banks (ξ = 0.45, 3 degree)



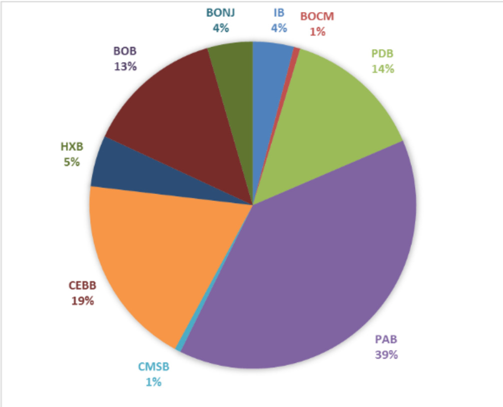
The 2 charts above shows that the distribution of banks' contribution to systemic risk remains stable as the critical threshold changes between the scopes.

Listed commercial banks

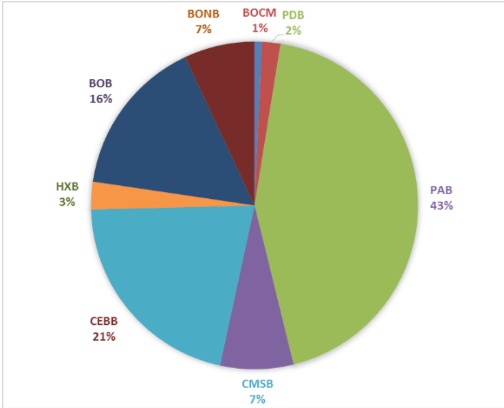
Same with the last section, Chart 5 shows 12 listed commercial banks' contribution to systemic risk with ξ = 0.3, 0.4 , 0.45 or 0.38.

Chart 5 Contribution of 12 listed commercial banks (with different ξ , 3 degree)

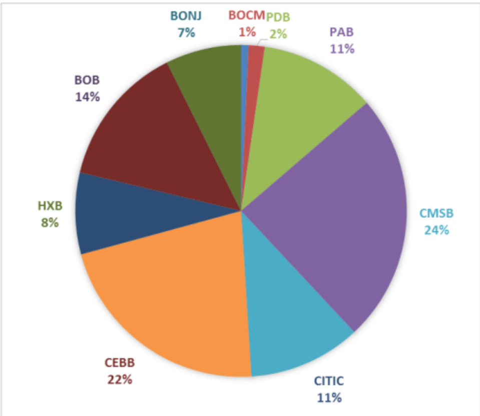
ξ = 0.3



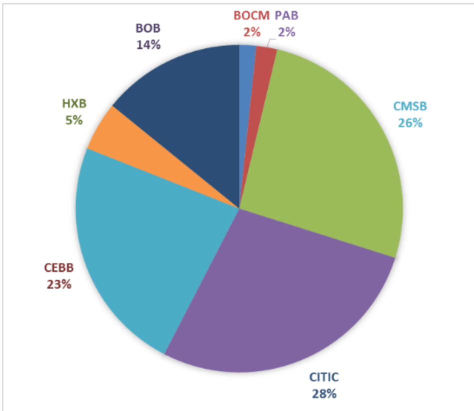
ξ = 0.38



ξ = 0.4



ξ = 0.45



Comparing the outcome with that of section 3.1.2, the distribution of contribution of ξ= 0.4 is still different from that of ξ= 0.42. However, they has been much more "similar" in the distribution, which may reflect that the distribution of the contribution of banks to systemic risk may become more stable in a relatively small interval with the classification of assets more and more elaborate. As we still cannot ignore the difference under current classification, some improvement or extensions are as follows.

3.2.1 Extensions - Asymmetric power indices

The approach can be extended to calculate asymmetric Shapley values, which include, the asymmetric power indices. A reason for doing this is that I want to incorporate the identity of other banks in a set of failed banks that triggers a systemic event into the risk measure, rather than only the bank in the set whose failure tips the system from stability to instability.

The next equation shows a modified characteristic function.

$$\begin{aligned} &\mathrm{v(G)} = 0 && \text{if } \Sigma_{i \in G} w_i \leq \xi' \\ &\mathrm{v(G)} = \mathrm{f}(\Sigma_{i \in G} w_i) && \text{if } \Sigma_{i \in G} w_i \in (\xi', \xi'') \\ &\mathrm{v(G)} = 0 && \text{if } \Sigma_{i \in G} w_i \geq \xi' \end{aligned} \tag{5}$$

where

$f' > 0$

and, f is such that $\mathrm{v(G)} \in (0,1)$ if $\Sigma_{i \in G} w_i \in (\xi', \xi'')$

With this function, the value of the characteristic function always lies between zero and one. If the set of bank failures is small enough (i.e. the proportion of failed banks' assets is less than or equal to ξ'), the value is zero (i.e. there is no systemic event). If the set of failed banks is large enough (i.e. the proportion of failed banks' assets is greater than or equal to ξ''), the value is one (i.e. there is a full systemic event). For intermediate levels (i.e. the proportion of failed banks' assets is between ξ' and ξ''), the value is between zero and one (i.e. there is a partial systemic event). Of course, the characteristic function nests the binary function used to calculate asymmetric power indices; by setting $\xi' = \xi''$, the characteristic function can be the same as in Section 2.

3.2.2 Extensions - The riskiness of assets

Differences in the riskiness of banks' exposures to an asset class could also be captured by extending the approach. It has to be considered that relative riskiness of asset holdings affects the order in which banks would fail for a given shock. Banks that hold more risky exposures to the asset classes should tend to be towards the front of the order of bank failures.

One way to reflect differences in the riskiness of exposures is to apply risk weights to banks' holdings of asset classes; the values of risk weights would be increasing in the riskiness of holdings. For example, risk weights r_i^h and r_i^f could be applied to bank's exposure to domestic and foreign assets, where $r_i^h, r_i^f \geq 0$, which mean bank 's position in the two-dimensional space becomes $a_i = (r_i^h h_i,\ r_i^f f_i) \in \mathbb{R}_+^2$.

4 Conclusions

This paper revealed that how the effect of bank failures on systemic risk is measured using the Shapley's asymmetric power index. It is calculated by identifying the prevalence with which each bank, in the event of its failure, would push the total assets of failed banks in the system beyond a critical threshold.

This paper reviews the Rodney's (2012) primary model of analyzing the identity of banks that are pivotal in the transition of a banking system from stability to instability at first. Since the general methodology of Rodney can be only applied to the banking system with limited number of banks and two-dimensional Euclidean leverage space, I created 2 new approaches using Matlab to break the limitation and extend the approach to higher degree like 3 or 5. Meanwhile, the new approach is applied to Chinese commercial banks, which means I have achieved the goal of using the approach as a tool for assessing the contributions of bank failures to systemic risk in practical banking system by and large. In addition, the paper evaluate the application effect of this approach finding its problem with the selection of the critical threshold when applied to the banking system with large number of banks. There is a relatively big difference between the distributions of banks' contributions to systemic risk with different critical thresholds in a small interval. To obtain a better application effect, two possible improvement or extensions are put forward: using asymmetric power indices and distinguishing riskiness of different banks' exposures. The approach in this paper can also be applied to reverse stress.